MA-161 (F,06)
Information & Formulas
For Test 2

GENERAL INSTRUCTIONS: Do all the problems on the paper I've provided unless otherwise directed. (Only Problem 8 is to be done on the test paper.) Be sure you number the pages and the problems clearly. Answer all problems in sentences, even those requiring a numerical or formulaic answer. (For example, if you were asked to differentiate $y = x^2$, your answer would be the sentence $y' = 2x$.) Be sure to show your work. Your work, as well as your answers, will be graded.

A number of formulas are on the Precalculus Formula Sheet, including the Factorization Formula and some exponent and log laws. Also, if you need a formula or can't remember how to do something, ask me and I'll (possibly) tell you.

Here are some of the derivative rules we derived: Assuming u is a function of x,

$\frac{d}{dx}(u^n) = n \cdot u^{n-1}\,\frac{du}{dx}$ $\quad \frac{d}{dx}(\sin u) = \cos u\,\frac{du}{dx}$ $\quad \frac{d}{dx}(\cos u) = -\sin u\,\frac{du}{dx}$ $\quad \frac{d}{dx}(\tan u) = \sec^2 u\,\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$ $\quad \frac{d}{dx}(\ln u) = \frac{1}{u}\cdot\frac{du}{dx}$ $\quad \frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}}\,\frac{du}{dx}$ $\quad \frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\,\frac{du}{dx}$ (See below.)

PLEASE: DO NOT FORGET TO USE THE CHAIN RULE, THE PRODUCT RULE, THE QUOTIENT RULE, ETC. WHEN THEY APPLY!

Also, you may use these special limits where necessary:

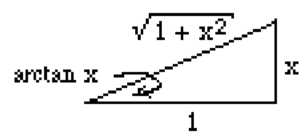
$\lim_{h\to 0}\frac{e^h - 1}{h} = 1$ $\quad \lim_{h\to 0}\frac{\sin h}{h} = 1$ $\quad \lim_{h\to 0}\frac{\cos h - 1}{h} = 0$ $\quad$ If P is a polynomial, then $\lim_{x\to a}P(x) = P(a)$.

Here is the derivation of the formula for $\frac{d}{dx}(\arctan x)$, using The Derivative of an Inverse Function Procedure:

Let $y = \arctan x$. Solve for x in terms of y to get $\tan y = x$. Differentiating both sides with respect to x, we get

$\frac{d}{dx}(\tan y) = \frac{d}{dx}(x)$. Since y is a function of x, the left side is $\sec^2 y\cdot\frac{dy}{dx}$ and the right side is 1.

$\sec^2 y\cdot\frac{dy}{dx} = 1$. Solving this equation for $\frac{dy}{dx}$, we get



$$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \left(\frac{1}{\sec y}\right)^2 = \left(\frac{1}{\sec(\arctan x)}\right)^2 = \left(\frac{1}{\sqrt{1+x^2}}\right)^2 = \frac{1}{1+x^2},$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Here is a derivation of the formula $\frac{d}{dx}(b^x) = b^x\cdot\ln b$ using Logarithmic Differentiation:

Let $y = b^x$. Then $\ln y = \ln b^x$ and $\ln y = x\cdot\ln b$. Differentiating both sides with respect to x, we get

$D_x(\ln y) = D_x(x\cdot\ln b)$. The left side is $\frac{1}{y}\cdot\frac{dy}{dx}$ and the right side is $\ln b$ (since ln b is a constant).

$\frac{1}{y}\cdot\frac{dy}{dx} = \ln b$. Solve for $\frac{dy}{dx}$ and replace y by what it's equal to to get

$\frac{dy}{dx} = y\cdot\ln b = b^x\cdot\ln b$. Consequently, $\frac{d}{dx}(b^x) = b^x\cdot\ln b$.